Formulas

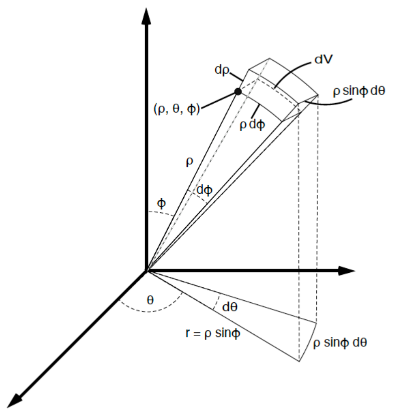
Regular

$$\vec{\nabla} f \neq 0$$

Cylindrical Coordinates

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z$$
$$dV = dz\,dA = r\,dr\,d\theta\,dz$$

Spherical Coordinates



$$r = \rho\sin\varphi, \quad x = r\cos\theta = \rho\sin\varphi\cos\theta,$$
$$y = r\sin\theta = \rho\sin\varphi\sin\theta, \quad z = \rho\cos\varphi$$
$$dV = \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

$: \rho \geq 0, 0 \leq \theta \leq 2\pi, 0 \leq \varphi \leq \pi$

Total Mass

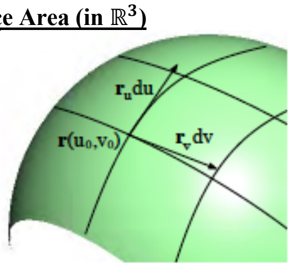
$$M = \iiint_T \delta(x,y,z)dV$$

Center of Mass

$$dm = \delta dV, \;\; M = \iiint_T dm$$
$$\bar{x} = \iiint_T x dm$$
$$\bar{y} = \iiint_T y dm$$
$$\bar{z} = \iiint_T z dm$$

- $r_{Centroid} \rightarrow \delta = 1$

Surface Area (in $\mathbb{R}^3$)



Parameterize Surface R using:
$$r(u,v) = (x(u,v), y(u,v), z(u,v))$$
$$SA = \iint_R dS = \iint_R |r_u \times r_v| du dv$$

If $r(x,y) = (x, y, f(x,y))$, then

$r_u \times r_v = (-f_x, -f_y, 1)$, and so
$$SA = \iint_R dS = \iint_R \sqrt{f_x^2 + f_y^2 + 1}\, du dv$$

Cross Product

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
$$(a,b,c) \times (c,d,e) = \begin{vmatrix} i & j & k \\ a & b & c \\ d & e & f \end{vmatrix} =$$
$$\vec{i}\begin{vmatrix} b & c \\ e & f \end{vmatrix} - \vec{j}\begin{vmatrix} a & c \\ d & f \end{vmatrix} + \vec{k}\begin{vmatrix} a & b \\ d & e \end{vmatrix}$$
$$|\vec{n} \times \vec{m}| = |\vec{n}||\vec{m}|\sin(\theta)$$

* Direction → right-hand rule*

$$n \times m = -m \times n$$
$$\vec{p} \times (a\vec{q} + b\vec{r}) = a(\vec{p} \times \vec{q}) + b(\vec{p} \times \vec{r})$$

Line Integral

Parameterize **C** using:

$r(t) = (x(t), y(t)), \quad t_1 \leq t \leq t_2$
$$\int_C F \cdot dr = \int_{t_1}^{t_2} F(r(t)) \cdot r'(t)dt$$

Conservative Vector Field

If $F(x,y,z) = \vec{\nabla} f(x,y,z)$,

F*(x,y,z)* → *conservative vector field*

f*(x,y,z)* → *potential function of* ***F***

- $\boldsymbol{curl(F) = curl(\vec{\nabla} f) = 0}$
- Fund. Th of Line Integrals

  If **C** is a curve from point **a** to point **b**
  $$\int_C F \cdot dr = f(\boldsymbol{b}) - f(\boldsymbol{a})$$

Green's Theorem

Given a region R bounded by a simple, closed curve ∂R (clockwise), and $\boldsymbol{F}(x,y) = (P(x,y), Q(x,y))$, then
$$\int_{\partial R} F \cdot dr = \iint_R curl(F)dA$$

Flux Through a Surface

Flux Integral through (outward) a region M:

Paramterize **M** using:

$r(u,v) = (x(u,v), y(u,v), z(u,v))$

n is the unit vector normal to the surface at a point

$\boldsymbol{n} = \frac{r_u \times r_v}{|r_u \times r_v|}, \quad \boldsymbol{dS} = |r_u \times r_v| du dv$
$$\iint_M \boldsymbol{F \cdot n dS} = \iint_M F(r(u,v)) \cdot \frac{r_u \times r_v}{|r_u \times r_v|} |r_u \times r_v| du dv$$
$$= \iint_M F(r(u,v)) \cdot (r_u \times r_v) du dv$$

The Divergence Theorem

∂R is a ccpr-surface, without boundary, bounding a region R (outwards):
$$\iint_{\partial R} \boldsymbol{F \cdot n dS} = \iiint_R div(F)dV$$

"The Flux through a boundary is the sum of all sources and sinks within the bounded region"

Stokes' Theorem

Given a ccpr-surface M with a boundary ∂M, oriented such that the surface is on the left of the positive direction of the curve, then
$$\int_{\partial M} F \cdot dr = \iint_M curl(F) \cdot ndS$$

Basic Derivatives

$$\frac{d}{dx}(f+g) = f' + g'$$
$$\frac{d}{dx}(f * g) = f'g + fg'$$
$$\frac{d}{dx}(f/g) = \frac{f'g - g'f}{g^2}$$
$$\frac{d}{dx}(x\char`^a) = ax^{a-1}$$
$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$
$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$$
$$\frac{d}{dx}(f(g(x)) = g' * f'(g)$$
$$\frac{d}{dx}(\sin x) = \cos x$$
$$\frac{d}{dx}(\cos x) = -\sin x$$
$$\frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$
$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

Trigonometric Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\sin(2\theta) = 2\sin(\theta)cos(\theta)$$
$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$
$$= 2\cos^2(\theta) - 1$$
$$= 1 - 2\sin^2(\theta)$$
$$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$$
$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\tan^2(\theta) = \sec^2(\theta) - 1$$

Chain Rule for Partial Derivatives

$$\frac{\partial f}{\partial t} = \vec{\nabla} f(\boldsymbol{x}) \cdot \frac{\partial \boldsymbol{x}}{\partial t}$$

Or
$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t} + \cdots$$

Divergence

$$div(F) = \vec{\nabla} \cdot F = \lim_{A_{(x,y)} \to 0} \frac{1}{|A_{(x,y)}|} \oint_C F \cdot \hat{n} dr$$

Curl

$$curl(F) = \vec{\nabla} \times F = \lim_{A_{(x,y)} \to 0} \frac{1}{|A_{(x,y)}|} \oint_C F \cdot dr$$